using integrated absorption coefficients for large line broadening ($\gamma \sim 0.5$ cm^{-1}) becomes a poor assumption, because significant absorption by the wings of the lines occurs outside the limits of the spectral range. For some of the same reasons, the results of line-by-line calculations cannot be accepted as a benchmark. Thus, the results of calculations using the direct simulation Monte Carlo method at this point must be considered as only approaching the correct results. To obtain more exact results it is necessary to either increase the number of simulated trajectories or to apply other methods.

The results for transmissivity have appreciable relative errors. This occurs because of the following reasons:

1) The solution of the transmissivity of selectively absorbing scattering layers requires special consideration (it is necessary to search for the optimum number of simulated trajectories, select a special simulation algorithm or another method, etc.).

2) The chosen number of simulated trajectories N_f does not ensure acceptable accuracy for a problem of this type. However, any further increase in N_f causes the algorithms to lose their basic advantage, high computational speed.

3) Despite appreciable errors, modifications of the simple Monte Carlo method are more economical than the method of line-by-line integration applied thousands or tens of thousands of times. Therefore it is useful to continue research such as this to find the most adequate and optimum methods.

Results for Emissivity

Calculation of emissivity is complicated because it is a selective function and, therefore, incurs additional errors from the averaging procedures.

Results for the hemispherical emissivity are presented in Table 2. The hemispherical emissivity for both surfaces is shown. For an exact solution, these values should be equal; the differences in the emissivities of the two surfaces serve as an empirical estimate of the accuracy of the results.

Again, accuracy is acceptable for optically thin media ($\tau_0 < 0.1$). In spite of the rather simple analysis needed for this case, the high accuracy in average emissivity is remarkable given the range of absorption coefficient of about three orders across the spectrum. Among the algorithms investigated, the most economic is the method of smoothing coefficients. In CPU time, the hybrid Monte Carlo method took about 10% longer, the two-group method about twice as long, and the method of line-by-line integration with few trajectories took many times longer. For large optical thickness ($\tau_0 > 10$) (not presented here), the hybrid method and method of smoothing coefficients experience errors of more than 100%, and the hybrid Monte Carlo method gives nonphysical results (emissivity of the layer is predicted to be more than unity). The main reason for this error is a poor choice of average energy of the photon groups by assuming optically thin emission. The method of line-by-line integration with few trajectories has the best accuracy.

Conclusions

Four approximate fast algorithms for direct Monte Carlo simulation that take into account the line structure of an absorption spectrum have been developed and investigated. It is shown that all algorithms allow calculation of radiation transfer in scattering volumes with sufficient accuracy for practical needs, particularly at optical thickness based on line centers of order unity.

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Analysis of Transient Heat Transfer in a Cylindrical Pin Fin

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Nomenclature

A	= dimensionless radius of fin, A^*/L
A^*	= radius of fin
Bi_a	= transversal Biot number, hA^*/k
Bi_T	= tip Biot number, $h_T L/k$
G	= geometry parameter for fin, L/A^*
H	= ratio of convective heat transfer coefficients, h_T/h
h	= convective heat transfer coefficient at lateral surface
h_T	= convective heat transfer coefficient at tip surface
J_n	= nth-order Bessel function
k	= thermal conductivity of fin
L	= fin length
$Q(t)$	= dimensionless transient heat flow at fin base, $Q_0(t^*)/2\pi A^* k(T_0^* - T_\infty^*)$
$Q_0(t^*)$	= transient heat flow at fin base
$T(t, x, r)$	= dimensionless transient temperature for two-dimensional solution, $T^*(t^*, x^*, r^*) - T_\infty^*/T_0^* - T_\infty^*$
$T^*(t^*, x^*, r^*)$	= transient temperature for two-dimensional solution
T_∞^*	= temperature of surroundings
T_0^*	= temperature at fin base
t	= dimensionless time, $\alpha t^*/L^2$
t^*	= time
x, r	= dimensionless coordinates, x^*/L, r^*/L
x^*, r^*	= axial and radial coordinates
α	= thermal diffusivity
β, η	= roots of transcendental equations

Introduction

THE use of extended surfaces or fins as heat transfer enhancement devices plays an important role in many thermal engineering applications. For one- or two-dimensional transient analysis of a fin, few papers have appeared in the literature. Chapman[1] first analyzed the transient response of an annular fin with uniform thickness subjected to a step change of base temperature. Yang,[2] Aziz,[3] and Suryanarayana[4] discussed the transient heat transfer problems with various base boundary conditions in a one-dimensional fin. Chu et al.[5] stud-

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ied the transient response of circular pin fins with a constant cross section subjected to a step change in base temperature or a step change in base heat flux in a two-dimensional analysis. However, these investigations are based on neglecting the tip effects in the one- or two-dimensional transient analysis. Mao and Rooke[6] analyzed the transient conduction in a one-dimensional rectangular fin with tip heat transfer using the method of inverse Laplace transforms.

The purpose of this study was to develop the analytical transient solutions using the Laplace transformation and separation of variables methods when the base is subjected to a step change in temperature, and the heat dissipation is convected from the lateral surface and the fin tip to the surroundings in a two-dimensional pin fin. The comparisons of error between the one- and two-dimensional transient heat flow rate at the base are presented with three tip convective conditions.

Analysis

The analysis is based on the following assumptions:

1) The convective heat transfer coefficients h and h_T, the temperature T_∞^* of the surrounding fluid, and the material properties of the pin fin are all assumed to be constant.

2) The pin fin is initially at the surrounding temperature T_∞^*.

3) The temperature distribution varies with t^*, x^*, and r^* in the two-dimensional pin fin, and varies with t^* and x^* in the one-dimensional pin fin.

4) There is no heat generation in the pin fin.

5) Radiation effects are neglected.

Two-Dimensional Solution

For a two-dimensional cylindrical pin fin, the governing equation and initial and boundary conditions in dimensionless form are

$$\frac{\partial T(t, x, r)}{\partial t} = \frac{\partial^2 T(t, x, r)}{\partial x^2} + \frac{\partial^2 T(t, x, r)}{\partial r^2} + \frac{1}{r}\frac{\partial T(t, x, r)}{\partial r} \tag{1}$$

$$t = 0, \qquad T(0, x, r) = 0 \tag{2}$$

$$t > 0, \qquad x = 0, \qquad T(t, 0, r) = 1 \tag{3}$$

$$x = 1, \qquad \frac{\partial T(t, 1, r)}{\partial x} + Bi_T T(t, 1, r) = 0 \tag{4}$$

$$r = 0, \qquad \frac{\partial T(t, x, 0)}{\partial r} = 0 \tag{5}$$

$$r = A, \qquad \frac{\partial T(t, x, A)}{\partial r} + G Bi_a T(t, x, A) = 0 \tag{6}$$

After utilizing the Laplace transformation technique and the method of separation of variables, the dimensionless temperature distribution for the analytical solution is derived as

$$\begin{aligned} T(t, x, r) = {} & \sum_{n=1}^{\infty} \frac{2Bi_a J_0(\eta_n G r)}{(\eta_n^2 + Bi_a^2) J_0(\eta_n)} \\ & \times \left\{ \frac{\eta_n G \cosh[\eta_n G(1 - x)] + Bi_T \sinh[\eta_n G(1 - x)]}{\eta_n G \cosh(\eta_n G) + Bi_T \sinh(\eta_n G)} \right. \\ & \left. - \sum_{m=1}^{\infty} \frac{2\beta_m^2 \exp[-(\beta_m^2 + \eta_n^2 G^2)^t] \sin(\beta_m x)}{(\beta_m^2 + \eta_n^2 G^2)[\beta_m - \cos(\beta_m)\sin(\beta_m)]} \right\} \end{aligned} \tag{7}$$

where η_n and β_m are the nth and mth positive roots of the following transcendental relations:

$$\eta_n J_1(\eta_n) - Bi_a J_0(\eta_n) = 0 \tag{8}$$

$$\beta_m \cot(\beta_m) = -Bi_T \tag{9}$$

The dimensionless transient heat flow rate at the fin base ($x = 0$) is given by

$$\begin{aligned} Q(t)_{2-D} = {} & \frac{Q_0(t^*)}{2\pi A^* k(T_O^* - T_\infty^*)} = -G \int_0^{1/G} \left. \frac{\partial T(t, x, r)}{\partial x} \right|_{x=0} r \, dr \\ = {} & \sum_{n=1}^{\infty} \frac{2Bi_a^2}{\eta_n^2 G(\eta_n^2 + Bi_a^2)} \\ & \times \left\{ \frac{\eta_n G[\eta_n G \sinh(\eta_n G) + Bi_T \cosh(\eta_n G)]}{\eta_n G \cosh(\eta_n G) + Bi_T \sinh(\eta_n G)} \right. \\ & \left. + \sum_{m=1}^{\infty} \frac{2\beta_m^3 \exp[-(\beta_m^2 + \eta_n^2 G^2)t]}{(\beta_m^2 + \eta_n^2 G^2)[\beta_m - \cos(\beta_m)\sin(\beta_m)]} \right\} \end{aligned} \tag{10}$$

where

$$Q_0(t^*) = -\int_0^{A^*} 2\pi r^* k \left. \frac{\partial T^*(t^*, x^*, r^*)}{\partial x^*} \right|_{x^*=0} dr^*$$

One-Dimensional Solution

For the case of a one-dimensional cylindrical pin fin, the solution obtained with the Laplace transformation method is

$$\begin{aligned} T(t, x) = {} & \left\{ \frac{G\sqrt{2Bi_a} \cosh[G\sqrt{2Bi_a}(1 - x)] + Bi_T \sinh[G\sqrt{2Bi_a}(1 - x)]}{G\sqrt{2Bi_a} \cosh(G\sqrt{2Bi_a}) + Bi_T \sinh(G\sqrt{2Bi_a})} \right. \\ & \left. - \sum_{m=1}^{\infty} \frac{2\beta_m^2 \exp[-(\beta_m^2 + 2Bi_a G^2)t] \sin(\beta_m x)}{(\beta_m^2 + 2Bi_a G^2)[\beta_m - \cos(\beta_m)\sin(\beta_m)]} \right\} \end{aligned} \tag{11}$$

The dimensionless transient heat flow at the base of the one-dimensional fin ($x = 0$) is

$$\begin{aligned} Q(t)_{1-D} = {} & \frac{Q_0(t^*)}{2\pi A^* k(T_O^* - T_\infty^*)} = -\frac{A}{2} \left. \frac{\partial T(t, x)}{\partial x} \right|_{x=0} \\ = {} & \frac{A}{2} \left\{ \frac{G\sqrt{2Bi_a}[G\sqrt{2Bi_a} \sinh(G\sqrt{2Bi_a}) + Bi_T \cosh(G\sqrt{2Bi_a})]}{G\sqrt{2Bi_a} \cosh(G\sqrt{2Bi_a}) + Bi_T \sinh(G\sqrt{2Bi_a})} \right. \\ & \left. + \sum_{m=1}^{\infty} \frac{2\beta_m^3 \exp[-(\beta_m^2 + 2Bi_a G^2)^t]}{(\beta_m^2 + 2Bi_a G^2)[\beta_m - \cos(\beta_m)\sin(\beta_m)]} \right\} \end{aligned} \tag{12}$$

where

$$Q_0(t^*) = -k\pi A^{*2} \left. \frac{\partial T^*(t^*, x^*)}{\partial x^*} \right|_{x^*=0}$$

To reveal the difference between one- and two-dimensional analytical transient solutions, the relative error of transient heat flow at the base is defined as

$$\text{ERR}(\%) = \left| \frac{Q(t)_{1-D} - Q(t)_{2-D}}{Q(t)_{2-D}} \right| \times 100\% \tag{13}$$

Results and Discussions

The solutions for T and Q depend on the parameters of H, G, and Bi_a because Bi_T can be expressed in terms of these parameters. To observe the individual effects of heat transfer and geometry characteristics, the tip Biot number Bi_T is given as follows:

$$Bi_T = HGBi_a$$

A Newton–Raphson method is used to find the roots of the transcendental equations (8) and (9). The first six values of the

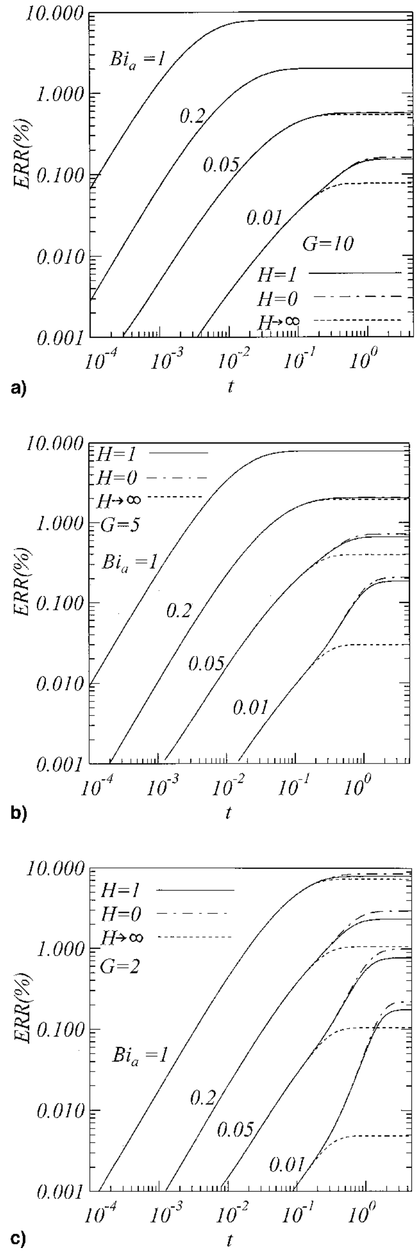

Fig. 1 Effects of Bi_a on relative error between one- and two-dimensional transient heat flow rate at the base. G = a) 10, b) 5, and c) 2.

results obtained in Eqs. (8) and (9) are in exact agreement with Tables III and II of Carslaw and Jaeger.[7] The convergence of the series' expressions for temperature distribution and heat flow is rather slow for small values of time ($t \leqq 0.01$). Therefore, it is required to determine the 250th root in Eq. (9). In each case, the summing process is terminated when the value of the last term computed is less than 0.000001.

Figures 1a–1c show that the relative errors of the base heat flow between one- and two-dimensional transient solutions are affected by H for different Bi_a at G = 10, 5, and 2. The relative errors increase with increasing Bi_a and t, and decrease with G at the fixed value of H. The steady-state condition is reached later as the values of G and Bi_a decrease. The relative error is insensitive to the variation in H for smaller times at the different Bi_a number. For fixed values of $Bi_a \leqq 0.2$, the effect of H on the relative error becomes more significant as G decreases at large times, but the effect of H on the relative error is insignificant when $Bi_a = 1.0$ and the different values of G. Inspection of Figs. 1a–1c reveals that the relative error is a minimum value as $H \to \infty$ and is a maximum value at $H = 0$ for the cases of $Bi_a = 0.01$ with various G values when steady state is reached. Similarly, this phenomenon is seen for the cases of $G = 2$ with various Bi_a values.

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Transient Temperature Analysis of Airplane Carbon Composite Disk Brakes

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Nomenclature

Bi = Biot number
h = convective heat transfer coefficient, W/m^2 K
k = thermal conductivity, W/mK
P = pressure, N/m^2
q'' = heat flux, W/m^2
R_i = inner radius of brake disk, m
R_o = outer radius of brake disk, m
T = temperature, K
t = time, s
z = axial coordinate, m
α = thermal diffusivity, m^2/s
δ = thickness of brake disk, m
ζ = dimensionless axial distance
θ = dimensionless temperature
μ = coefficient of friction
ρ = dimensionless radial distance
ρC = heat capacity, J/m^3 K
τ = dimensionless time
ω = angular speed, 1/s

Introduction

BECAUSE of outstanding high-temperature heat dissipating qualities, carbon–carbon advanced composites have become attractive materials for aircraft disk brake pads. The use of chemical vapor deposition (CVD) techniques in the preparation of composite structures has made considerable improvements possible in thermomechanical properties of carbon–carbon composite materials.[1] Carbon has a high heat ab-

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